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MEASUREMENT OF 3-D MOTION

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General

The general three-dimensional (3-D) motion of a rigid body (R.B.), described by its six degrees of freedom, may be determined by means of nine linear (translational) accelerometers which are properly mounted on the R.B. in three clusters. The nine accelerations are mathematically manipulated to produce the angular acceleration and velocity vectors of the R.B., its orientation in 3-D inertial (laboratory) space, the absolute translational acceleration, velocity and position vectors of an arbitrary point on the rigid body.

Each one of the three clusters is a triaxial accelerometer (or triax) which measures 3 orthogonal components of the acceleration vector of its "center." In general, these orhogonal instrumentation axes are arbitrarily chosen for convenience, and threfore, do not coincide with the "anatomical" reference frame in which the motion is to be described. It is assumed that the transformation matrix between the instrumentation and anatomical frames is known, so that the analysis is conducted on the anatomical components of the three acceleration vectors. Furthermore, it is assumed that the position vector of each triax center is known relative to an arbitrary point, which is taken as the origin of the anatomical reference frame.

In the following sections, the kinematic equations of motion will be developed, their solution will be presented, then validated with actual and hypothetical motions.

Kinematics Of Angular Motion

Consider the rigid body shown in figure 1. The anatomical reference frame ($\underline{i},\underline{j},\underline{k}$) is used to express the position vectors of the triax centers Q1, Q2 and Q3 relative to an arbitrary reference point QØ:

$$\underline{P1} = PI1\ \underline{i} + PJ1\ \underline{j} + PK1\ \underline{k}$$

$$\underline{P2} = PI2\ \underline{i} + PJ2\ \underline{j} + PK2\ \underline{k} \qquad (1)$$

$$\underline{P3} = PI3\ \underline{i} + PJ3\ \underline{j} + PK3\ \underline{k}$$

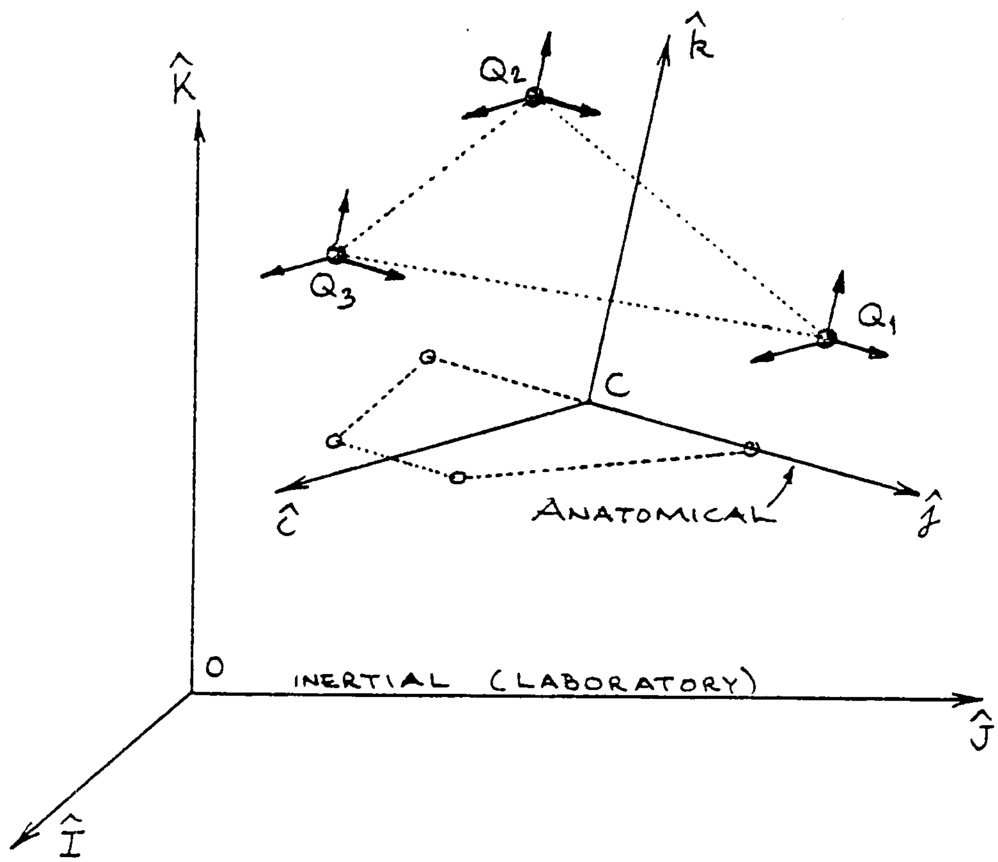


FIGURE 1. COORDINATE SYSTEMS AND THE NINE ACCELERATIONS USED IN 3-D MOTION ANALYSIS.

The measurement of nine accelerations defines the acceleration vectors at Q1, Q2 and Q3 which may be expressed in the anatomical frame:

$$\underline{A1} = AI1\ \underline{i} + AJ1\ \underline{j} + AK1\ \underline{k}$$
$$\underline{A2} = AI2\ \underline{i} + AJ2\ \underline{j} + AK2\ \underline{k} \qquad (2)$$
$$\underline{A3} = AI3\ \underline{i} + AJ3\ \underline{j} + AK3\ \underline{k}$$

The same acceleration vectors, given in equation (2), may be calculated from the kinematic equation of motion of a rigid body, given the translational acceleration vector of a body reference point and the angular acceleration (and velocity) vectors of the R.B.:

$$\underline{A1} = \underline{A} + \underline{U}x\underline{P1} + \underline{W}x\underline{W}x\underline{P1}$$
$$\underline{A2} = \underline{A} + \underline{U}x\underline{P2} + \underline{W}x\underline{W}x\underline{P2} \qquad (3)$$
$$\underline{A3} = \underline{A} + \underline{U}x\underline{P3} + \underline{W}x\underline{W}x\underline{P3}$$

where $\underline{U}$, $\underline{W}$ and $\underline{A}$ are angular acceleration, velocity and translational acceleration vectors, respectively. These three vectors are unknown and must be determined as:

$$\underline{A} = AI\ \underline{i} + AJ\ \underline{j} + AK\ \underline{k} \qquad (4)$$

$$\underline{U} = UI\ \underline{i} + UJ\ \underline{j} + UK\ \underline{k} \qquad (5)$$

$$\underline{W} = WI\ \underline{i} + WJ\ \underline{j} + WK\ \underline{k} \qquad (6)$$

Vector-equation (3) represents 9 scalar equations, in which the unknowns are the components of equations (4), (5) and (6). Note however that the angular velocity is simply the integral of the angular acceleration, which means that there is a third-order redundancy in these equations.

Least-Squares Solution

The first step towards the solution of equation (3), is to write it in matrix form. The position vectors $\underline{P1}$, $\underline{P2}$ and $\underline{P3}$ are replaced by column matrices {P1}, {P2} and {P3} given by:

$$\{P1\} = \begin{Bmatrix} PI1 \\ PJ1 \\ PK1 \end{Bmatrix} \qquad \{P2\} = \begin{Bmatrix} PI2 \\ PJ2 \\ PK2 \end{Bmatrix} \qquad \{P3\} = \begin{Bmatrix} PI3 \\ PJ3 \\ PK3 \end{Bmatrix}$$

The acceleration vectors $\underline{A1}$, $\underline{A2}$ and $\underline{A3}$ are replaced by

$$\{A1\} = \begin{Bmatrix} AI1 \\ AJ1 \\ AK1 \end{Bmatrix} \qquad \{A2\} = \begin{Bmatrix} AI2 \\ AJ2 \\ AK2 \end{Bmatrix} \qquad \{A3\} = \begin{Bmatrix} AI3 \\ AJ3 \\ AK3 \end{Bmatrix}$$

The angular acceleration $\underline{U}$ and velocity $\underline{W}$ vectors are also replaced by the column matrices $\{U\}$ and $\{W\}$:

$$\{U\} = \begin{Bmatrix} UI \\ UJ \\ UK \end{Bmatrix} \qquad \{W\} = \begin{Bmatrix} WI \\ WJ \\ WK \end{Bmatrix}$$

The cross-products, of the type $\underline{PxU}$ and $\underline{WxP}$, given in equation (3) are re-written by introducing the skew-symmetric matrices [W], [P1], [P2] AND [P3] defined by:

$$[W] = \begin{bmatrix} \emptyset & -WK & WJ \\ WK & \emptyset & -WI \\ -WJ & WI & \emptyset \end{bmatrix} \quad \text{and} \quad [P] = \begin{bmatrix} \emptyset & -PK & PJ \\ PK & \emptyset & -PI \\ -PJ & PI & \emptyset \end{bmatrix}$$

It may be shown that a cross-product of two vectors is equivalent to the product of the skew-symmetric matrix of the first, times the colmun matrix of the second. With these definitions and properties, equation (3) may be re-written in the following form:

$$\begin{aligned} \{A\} - [P1]\{U\} + [W]\{\ [W]\{P1\}\ \} - \{A1\} &= \emptyset \\ \{A\} - [P2]\{U\} + [W]\{\ [W]\{P2\}\ \} - \{A2\} &= \emptyset \qquad (7) \\ \{A\} - [P3]\{U\} + [W]\{\ [W]\{P3\}\ \} - \{A3\} &= \emptyset \end{aligned}$$

which may further be combined in a single equation:

$$\{a\} - [p]\{U\} + \{v\} = \emptyset \qquad (8)$$

where $\{a\}$ and $\{v\}$ are 9x1 column matrices, and [p] is a 9x3 matrix, defined as follows:

Least-Squares Solution

$$\{a\}=\begin{Bmatrix}\{A\}\\ \{A\}\\ \{A\}\end{Bmatrix} \qquad [p]=\begin{bmatrix}[P1]\\ [P2]\\ [P3]\end{bmatrix} \quad \text{and} \quad \{v\}=\begin{Bmatrix}[W]\{[W]\{P1\}\}-\{A1\}\\ [W]\{[W]\{P2\}\}-\{A2\}\\ [W]\{[W]\{P3\}\}-\{A3\}\end{Bmatrix}$$

Equation (8) is a set of nine scalar simultaneous equations in six unknowns. The redunduncy may be eliminated by realizing that, because of the experimental errors inevitably present in the nine acceleration measurements, the left-hand side of this equation is never exactly zero, but rather a small error 9x1 matrix {e}:

$$\{a\} - [p]\{U\} + \{v\} = \{e\} \tag{9}$$

In order to minimize the measurement-induced errors, consider the sum of squares of these errors, given by the quadratic form {e}'[c]{e}, where {e}' is the transpose of {e}, and [c] is the matrix of co-variances. It may be assumed that the errors are independant and have equal variances; then, the matrix [c] may be replaced by the unit matrix, and the quadratic form in question becomes {e}'{e}.

This square error will have a minimum when its partial derivatives with respect to the six unknowns are zero; i. e., when

$$d(\ \{e\}'\{e\}\ )/d\{U\} = 0 \tag{10}$$

$$d(\ \{e\}'\{e\}\ )/d\{A\} = 0 \tag{11}$$

where "d" denotes a partial differential operator. It turns out that it is not necessary to expand equation (11), since the same equations resulting from the expansion are obtained directly by another approach, as will be shown later.

Equation (10) may be expanded as

$$(\ d\{e\}'/d\{U\}\ )\{e\} + \{e\}'(\ d\{e\}/d\{U\}\ ) = 0$$

which may be shown to be equivalent to

$$2(\ d\{e\}'/d\{U\}\ )\{e\} = 0 \tag{12}$$

Using the prime (') to indicate the transpose of a matrix, the partial derivative of {e}' with respect to {U} becomes:

$$d(\ \{a\} - [p]\{U\} + \{v\}\ )'\ /\ d\{U\} = -[p]' \tag{13}$$

which can be substituted in equation (12):

$$-[p]'(\ \{a\} - [p]\{U\} + \{v\}\ ) = 0$$

or the final form

$$-[p]'\{a\} + [p]'[p]\{U\} - [p]'\{v\} = 0 \qquad (14)$$

This final form involves {A} and {U} as unknowns. It is possible to follow a similar procedure by differentiating with respect to {A} to obtain another set of equations; however, the procedure is greatly simplified if the reference point Q0 is selected to be the centroid of Q1, Q2 and Q3. One such simplification is that the unknown accceleration vector {A} is directly obtained from

$$\{A\} = (\ \{A1\} + \{A2\} + \{A3\}\ ) / 3 \qquad (15)$$

Another simplification is that the first term of equation (14) drops out leaving only three unknowns:

$$[p]'[p]\ \{U\} - [p]'\ \{v\} = 0 \qquad (16)$$

which may be solved for {U} by pre-multiplying it by the inverse of the {U} factor, resulting in

$$\{U\} = [\ [p]'[p]\ ]^{-1}[p]'\ \{v\} \qquad (17)$$

This equation is a set of three simultaneous differential equations in which the unknowns are the angular velocity components WI, WJ and WK and their derivatives the acceleration components UI, UJ and UK. All other terms in this equation are either constant or known from accelerometer measurements. The solution may be obtained by numerical integration and the resulting angular accelerations and velocities are the best estimates, in the least-squares sense, of the true solution.

Angular Motion

Once the angular acceleration and velocity vectors are computed, the next phase is to define the orientation in inertial (laboratory) reference frame of the rigid body. This may be done by determining three Euler angles as functions of time. These angles define a direction cosines matrix which describes the rotation of the anatomical frame (i,j,k) relative to an initial orientation coinciding with the laboratory frame (I,J,K):

$$\{\ \underline{i}\ \underline{j}\ \underline{k}\ \} = [E]\ \{\ \underline{I}\ \underline{J}\ \underline{K}\ \} \tag{18}$$

The three Euler angles are defined as follows: The first rotation is a yaw (PSI) about the initial $\underline{K}$-axis, resulting in an intermediate frame. The second rotation is a pitch (THETA) about the new $\underline{J}$-axis, resulting in yet another new frame. The last rotation is a roll (PHI) of this last frame about its $\underline{I}$-axis. This results in the final ($\underline{i}$,$\underline{j}$,$\underline{k}$) frame. To define the matrix [E], let

S1 = sin(PSI)   S2 = sin(THETA)   S3 = sin(PHI)

C1 = cos(PSI)   C2 = cos(THETA)   C3 = cos(PHI)

and let "*" denote a scalar multiplication; then,

$$[E] = \begin{bmatrix} C1*S2 & S1*C2 & -S2 \\ C1*S2*S3-S1*C3 & S1*S2*S3+C1*C3 & C2*S3 \\ C1*S2*C3+S1*S3 & S1*S2*C3-C1*S3 & C2*C3 \end{bmatrix} \tag{19}$$

In order to solve for the three Euler angles, consider their time-derivatives, denoted by W1, W2 and W3. It may be shown that the angular velocity vector $\underline{W}$, obtained from the solution of equation (17), is the sum of three angular rate vectors which have magnitudes of W1, W2 and W3, and which may or may not be orthogonal:

$$\underline{W} = \underline{W1} + \underline{W2} + \underline{W3} \tag{20}$$

Equation (20) may be projected on the three moving anatomical axes ($\underline{i}$,$\underline{j}$,$\underline{k}$) so that the anatomical components WI, WJ, and WK are written as sums of contributions from W1, W2 and W3:

$$\begin{Bmatrix} WI \\ WJ \\ WK \end{Bmatrix} = \begin{bmatrix} -S2 & 0 & 1 \\ C2*S3 & C3 & 0 \\ C2*C3 & -S3 & 0 \end{bmatrix} \begin{Bmatrix} W1 \\ W2 \\ W3 \end{Bmatrix} \tag{21}$$

The unknowns in this equation are the Euler angles, represented by their sines and cosines, and their rates W1, W2 and W3, while the anatomical components WI, WJ and WK of the angular velocity are known. Solving for the unknowns yield the following differential equation:

$$\begin{Bmatrix} W1 \\ W2 \\ W3 \end{Bmatrix} = (1/C2) \begin{bmatrix} 0 & S3 & C3 \\ 0 & C3 & -S3 \\ 1 & S2*S3 & S2*C3 \end{bmatrix} \cdot \begin{Bmatrix} WI \\ WJ \\ WK \end{Bmatrix} \qquad (22)$$

Equation (22) may be numerically integrated to yield the three Euler angles and their rates. Note that no solution exists when C2=0, i.e., when the pitch THETA is exactly 90 degrees. This case, known as the "Gimbal lock," may be avoided by either switching to another set of Euler angles, or by a numerical algorithm which "jumps" over this critical value.

By solving for the Euler angles, the three rotational degrees of freedom are completely specified.

Translational Motion

The next and final phase of 3-D motion analysis is to determine the translational acceleration, velocity and position vectors of an arbitrary point on the R.B. with respect to the inertial laboratory ($\underline{I}$,$\underline{J}$,$\underline{K}$) reference frame. The first step is to determine the laboratory components XDD, YDD and ZDD of the acceleration vector of body-point B, then integrate them once to obtain the velocity components XD, YD and ZD, which may be integrated again to obtain the laboratory coordinates X, Y and Z of the desired body-point B.

Consider an arbitrary point B on the rigid body whose coordinates with respect to the moving anatomical frame are contant and known:

$$\underline{PB} = PBI\ \underline{i} + PBJ\ \underline{j} + PBK\ \underline{k} \qquad (23)$$

Given the angular accelration $\underline{U}$ of the R.B., its angular velocity $\underline{W}$, and the acceleration $\underline{A}$ of the reference point Q0 (taken as origin,) all expressed in the anatomical frame, the absolute acceleration of point B is given by:

$$\underline{AB} = \underline{A} + \underline{U}x\underline{PB} + \underline{W}x\underline{W}x\underline{PB} \qquad (24)$$

When this equation is expanded, the components ABI, ABJ and ABK of the acceleration $\underline{AB}$, along the anatomical frame, are determined, so that

$$\underline{AB} = ABI\ \underline{i} + ABJ\ \underline{j} + ABK\ \underline{k} \tag{25}$$

This same vector may be written in terms of the laboratory ($\underline{I}$,$\underline{J}$,$\underline{K}$) reference frame as

$$\underline{AB} = XDD\ \underline{I} + YDD\ \underline{J} + ZDD\ \underline{K} \tag{26}$$

where XDD, YDD and ZDD are to be determined. To compute these unknowns, recall that, since the Euler angles have been determined, the transformation matrix [E] given by equation (19) is completely determined; thus, the unit vectors $\underline{i}$, $\underline{j}$ and $\underline{k}$ may be expressed in terms of the $\underline{I}$, $\underline{J}$ and $\underline{K}$ vectors, using equation (18). Substituting these expressions in equation (25) yields

$$\begin{Bmatrix} XDD \\ YDD \\ ZDD \end{Bmatrix} = \{\ ABI\ ABJ\ ABK\ \}\ [E] \tag{27}$$

Once the time histories of XDD, YDD and ZDD have been calculated, the absolute velocities XD, YD and ZD may be obtained by simple integrations. Finally, these are in turn integrated to give the laboratory coordinates X, Y and Z of the body-point B in question.

With this, the three translational degrees of freedom of the rigid body are completely specified.

Input Requirements

The 3-D analysis presented in the previous sections is carried out at HSRI by the computer program "NINACC" which calls several subroutines to perform the various phases of the analysis. The current version of "NINACC" is interactive with the user who can control the progress of the analysis from a computer terminal.

Input to the program is prepared ahead of run time and stored on disk and digital tape files. At run time, the program prompts the user to specify the locations of the various input data, and after reading it, asks for verification and/or modification. The input data may be divided according to the method and source of preparation into three groups.

GROUP 1: The first group of data is required to transform the acceleration data from the instrumentation reference frame to the anatomical one. It consists of the 9-element orthogonal transformation matrix, the anatomical coordinates of the origin of the instrumentation frame, the locations of the three triaxial accelerometers centers in the instrumentation frame, and the anatomical coordinates of an arbitrary body-point whose translational motion is to be studied.

GROUP 2: This group consists of 4 sets (3 components each) of the initial values of the anatomical angular velocity, the inertial (laboratory-referred) Euler angles, the inertial translational velocity of the body-point of interest, and the corresponding position.

GROUP 3: This group is the time-histories of 9 accelerometers readings. These are originally analog transducer outputs which have been converted to digital signals. It is necessary to ensure that the analog-to-digital conversion process and the filtering of the signals (analog and digital) do not introduce any phase shift between one signal and the others.

The first two groups are usually obtained from high-speed movies and x-ray films. The methods used must produce three-dimensional orthogonal components of velocities and positions. Such methods have been developped at HSRI and their applications have produced satisfactory results.

Programming Considerations

The 3-D analysis program "NINACC" calls special subroutines to perform the various phases of input, computations, and output. These subroutines are listed below, along with brief descriptions of the functions they perform.

1. NINACC Controls the calling sequence.

2. X9RCON Reads the two groups of constants described in the pervious section.

3. X9RACC Reads the digitized signals of 9 accelerometers.

4. X9PREP Transforms all data into the anatomical frame; prepares the constant matrix of equation (17); computes the acceleration of the reference point Q0, given by equation (15); outputs the raw 9 acceleration readings.

5. C91000 Calls "SXHPCG" to integrate the angular accelerations given by equation (17), which calls at each time step "C9FCT" to compute the right-hand side of the equation, integrates one time step, then returns the integrated values to "C91OUT" to record them; outputs the time-histories of angular motion when integration is completed.

6. X92000 Calls "SHPCG" to integrate the Euler rates, given by equation (22), which in turn calls "C92FCT" and "C92OUT"; outputs the Euler rates and angles when integration is completed.

7. X93000 Selects a body-point with user's help; calculates the anatomical accelerations of that point; calculates the inertial components using equation (27); calculates the Gadd Severity Index (GSI) and the Head Injury Criterion (HIC) numbers; outputs the results of calculations; and repeats this procedure for another body-point.

8. X94000 Integrates the inertial accelerations of the last body-point selected above, using "SXHPCG", "X94FCV" and "X94OUV"; integrates the resulting velocities using "SXHPCG", "9X4FCP" and "9X4OUP"; outputs the translational velocities and positions.

The numerical integration routine "SXHPCG" called above is a modified version of the IBM routine "HPCG". The modifications were necessary to force output at specified and equal intervals of time, and to simplify the list of input arguments. The numerical method used for integeration is best decribed by quoting from the "HPCG" description, given in the IBM System/360 Scientific Subroutine Package, Version III, publication number GH 20-0205-4:

> "These subroutines use Hamming's modified predictor-corrector method for the solution of general initial-value problems. ...to obtain an approximate solution of a general system of first-order ordinary differential equations with given initial values. It is a stable fourth-order integration procedure that requires the evaluation of the right-hand side of the system only two times per step. This is a great advantage compared to other methods of the same order of accuracy, especially the Runge-Kutta method, which requires the evaluation of the right-hand side four times per step. Another advantage is that at each step the calculation procedure gives an estimate for the local truncation error; thus, the procedure is able, without significant amount of calculation time, to chose and change the step size h. On the other hand, Hamming's predictor-corrector method is not self-starting; that is, the functional values at a single previous point are not enough to get the functional values ahead. Therefore, to obtain the starting values, a special Runge-Kutta procedure followed by one iteration step is added to the predictor-corrector method."

Except for the SSP routine "HPCG," all subroutines were developped at HSRI and coded in Fortran-IV to be run on MTS, the Michigan Terminal System. Therefore, many input/output subroutines are system-dependant and are designed to process data generated at HSRI and make use of MTS support routines; However, the final output is saved on unlabeled magnetic tape in card image (EBCDIC characters,) and may be read and post-processed at any computer installation.

Validation: Hypothetical Motion

The 3-D motion measurement method, described in the previous sections, may be validated by comparing the calculated motion against what is known to be the true one. The procedure is to generate nine acceleration readings resulting from a precisely known motion, and "feed" them to the analysis program "NINACC" which will predict the 3-D motion. It is difficult to generate a precisely known general 3-D motion in the laboratory; however, it is simpler to simulate such a motion where all 6 degrees of freedom are variable functions of time. These may be prescribed by closed-form mathematical functions, which are at least twice-differentiable.

This section describes a validation procedure in which a hypothetical motion is imposed on a hypothetical rigid body, by specifying closed-form functions for the 3 rotations and 3 translations. The 3-D analysis is performed in reverse order to generate exact time-histories of various quantities that the "NINACC" will generate. Knowing the exact motion, the readings of nine hypothetical accelerometers are simulated by computing the acceleration vectors at their centers. These "readings" are then used as input to "NINACC" analysis program to compute the motion, which is finally compared to the exact one.

The mathematical function selected to describe each of the six degrees of freedom has the general form:

$$S(t) = S_m \left[\sin 2\pi f_1 t - \sin 2\pi f_2 t\right]$$

where f_1 and f_2 are frequencies (Hertz) of the two sinusoidal components, and S_m is the amplitude (inches or radians) of either component. This form is continuous, and has a continuous first time-derivative (velocity) given by

$$\dot{S}(t) = 2\pi S_m \left[f_1 \cos 2\pi f_1 t - f_2 \cos 2\pi f_2 t\right]$$

This velocity is also continuous, and has an acceleration which is zero at t=0, given by

$$\ddot{S}(t) = -4\pi^2 S_m \left[f_1^2 \sin 2\pi f_1 t - f_2^2 \sin 2\pi f_2 t\right]$$

To simulate a general 3-D motion, 6 amplitudes and 12 frequencies must be specified. The simulation presented in this validation is based on the values given in table 1, which completely specify the rigid body motion. To generate the 9 acceleration readings, the distances R1, R2 and R3 of the triaxial centers to the origin were given as 3.0, 4.0 and 5.0 inches, respectively. Finally, a sampling rate of 1600. Hertz was specified to generate tabular time-histories of these signals. This rate must be at least twice the highest frequency contained in the digitized signals.

	TRANSLATIONS			ROTATIONS		
	X	Y	Z	YAW	PITCH	ROLL
Amplitude:	25.	7.	20.	20.	15.	25.
1st Freq.:	2.0	1.0	4.0	12.	7.0	8.0
2nd Freq.:	3.5	5.0	2.5	4.0	13.	0.5

Table 1. Amplitudes (Inches or Degrees) and Frequencies (Hertz) of the Six Degrees of Freedom of the Simulated 3-D Motion.

Once the positions, velocities and accelerations of the 3 translations and 3 rotations have been specified, as in table 1, the various time-histories are computed and saved in appropriate files for later comparisons. Thus, the translational positions and velocities of the origin, in the inertial (laboratory) frame, are saved as shown in figure 2. Given the Euler angles and the inertial translational accelerations of the origin, its anatomical components may be computed, and both types are saved in one file, which is plotted in figure 3. The Euler angles and their rates, shown in figure 4, are saved in a third file. These are used to compute the angular velocities and accelerations which are also saved and plotted in figure 5. Finally, given the complete description of motion, and the distances R1, R2 and R3, the "readings" of the 9 accelerometers are computed, and are shown in figure 6.

Given the 9 accelerometer "readings" and their locations and orientations on the rigid body, the "NINACC" program was called to perform the 3-D motion analysis. The program then produced 4 output files containing the various groups of variables describing the 3-D motion. These files were plotted and compared to the exact time-histories described in the previous paragraph. The various plots were so similar that it was not possible to distinguish graphically between the exact and calculated ones. Therefore, plots of the calculated motion are not shown here. Instead, the differences between calculated and exact motions were calculated point-by-point and plotted in figures 7 - 10.

Validation: Hypothetical Motion

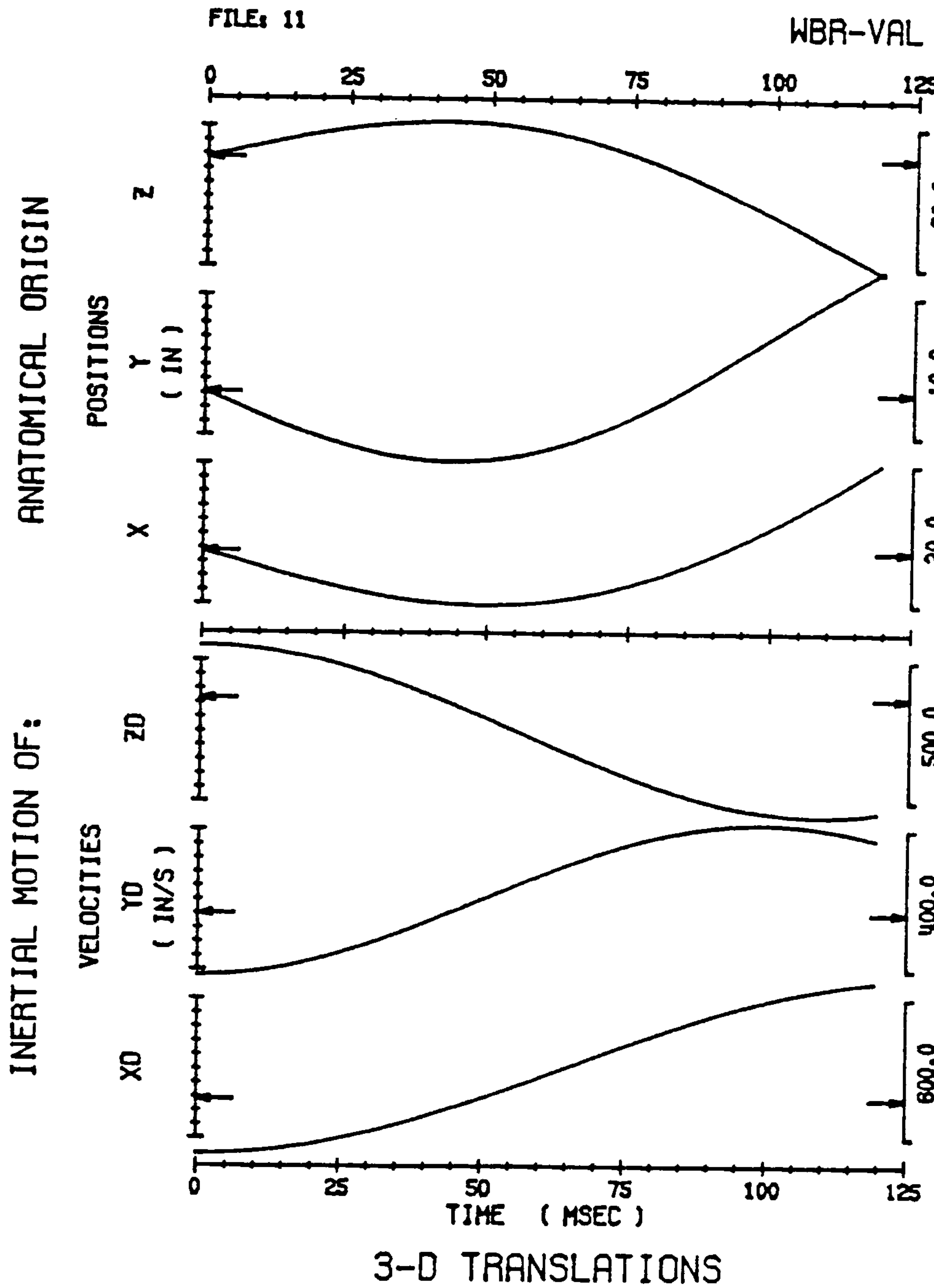


FIGURE 2. TRANSLATIONAL POSITIONS AND VELOCITIES OF THE ORIGIN OF THE SIMULATED 3-D MOTION.

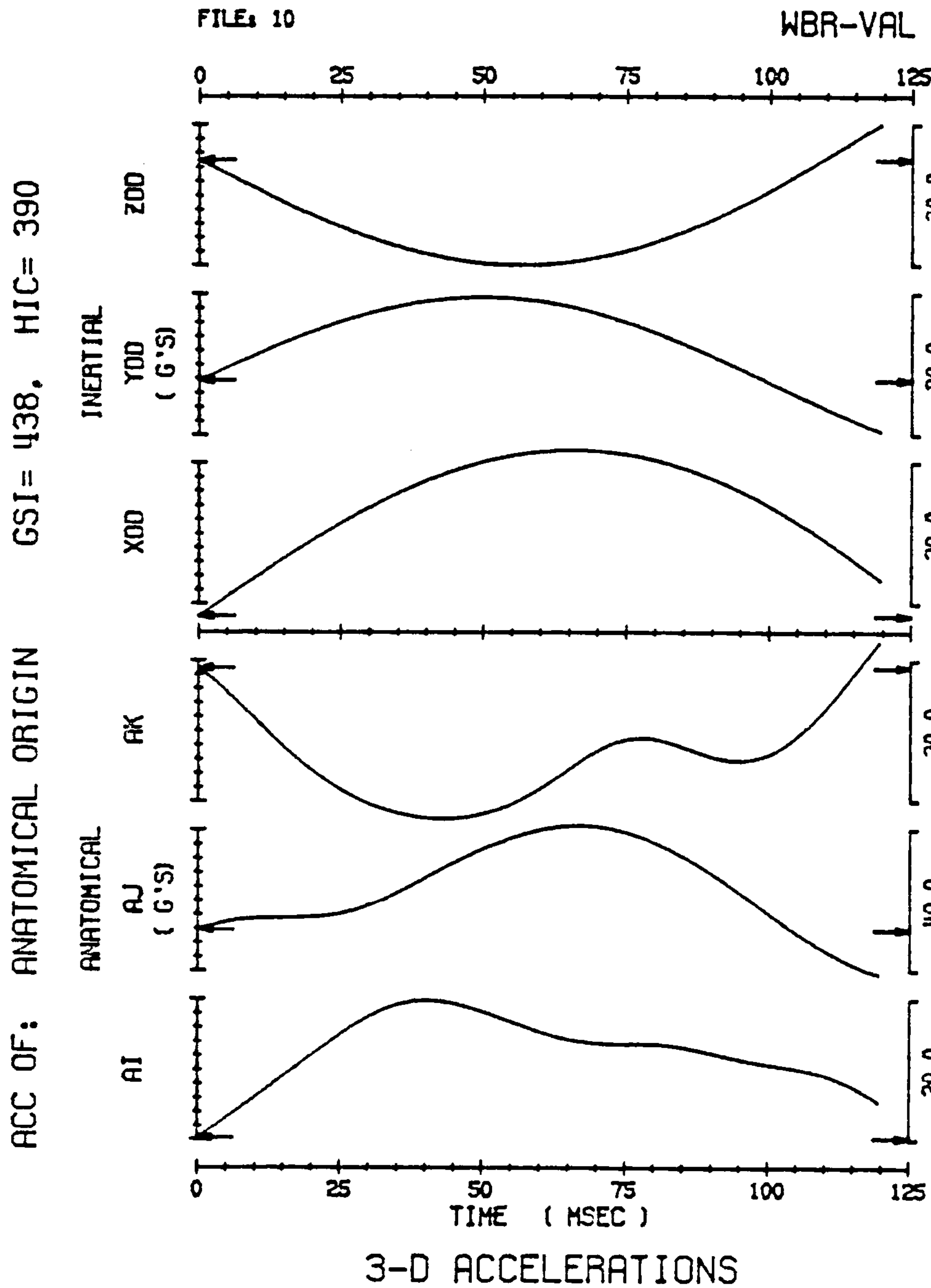


FIGURE 3. HYPOTHETICAL TRANSLATIONAL ACCELERATIONS OF ORIGIN IN INERTIAL(FIXED) AND ANATOMICAL(MOVING) FRAMES.

Validation: Hypothetical Motion

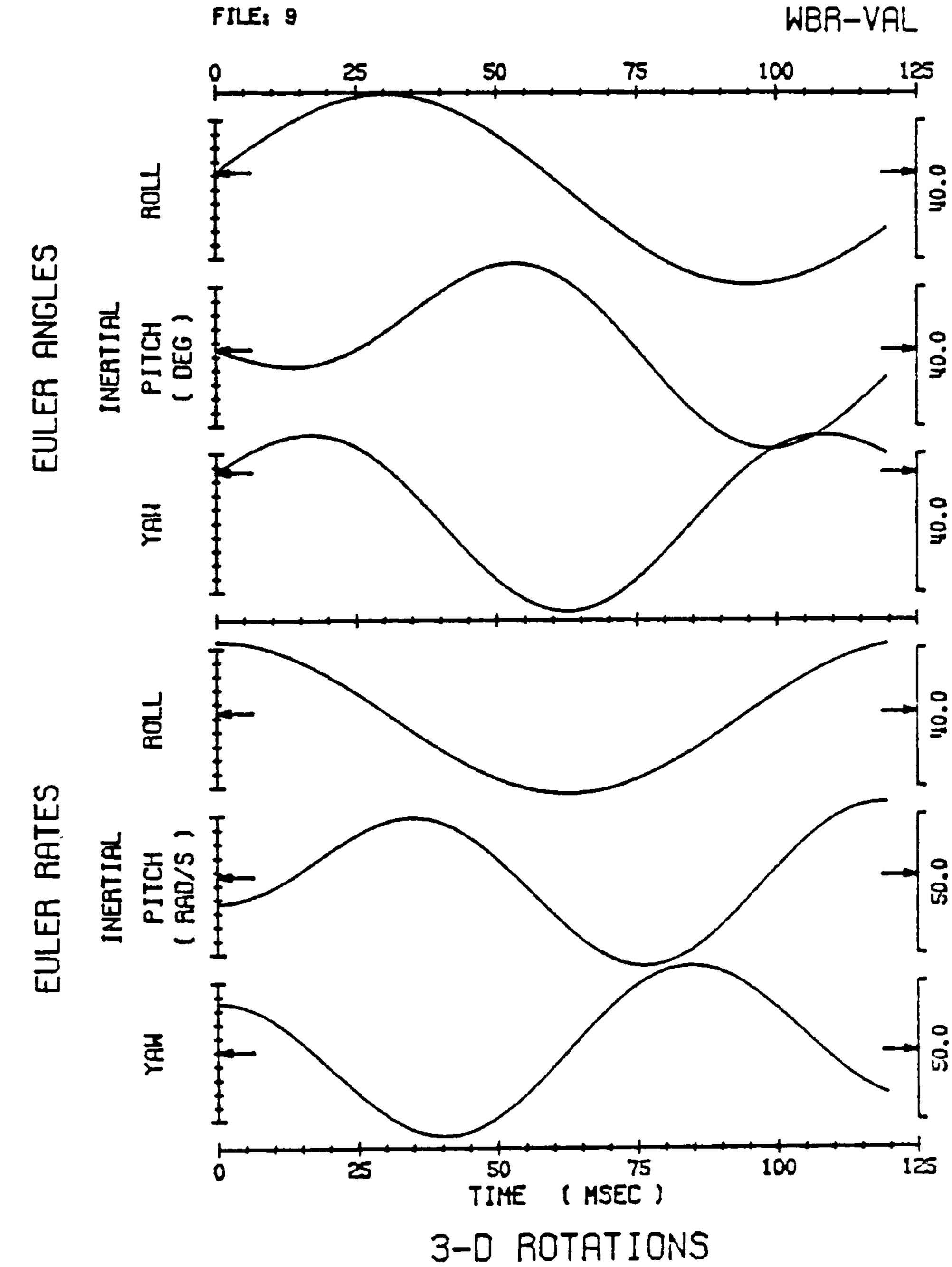


FIGURE 4. EULER ANGLES AND THEIR RATES FOR THE HYPOTHETICAL 3-D MOTION, GIVEN IN INERTIAL FRAME.

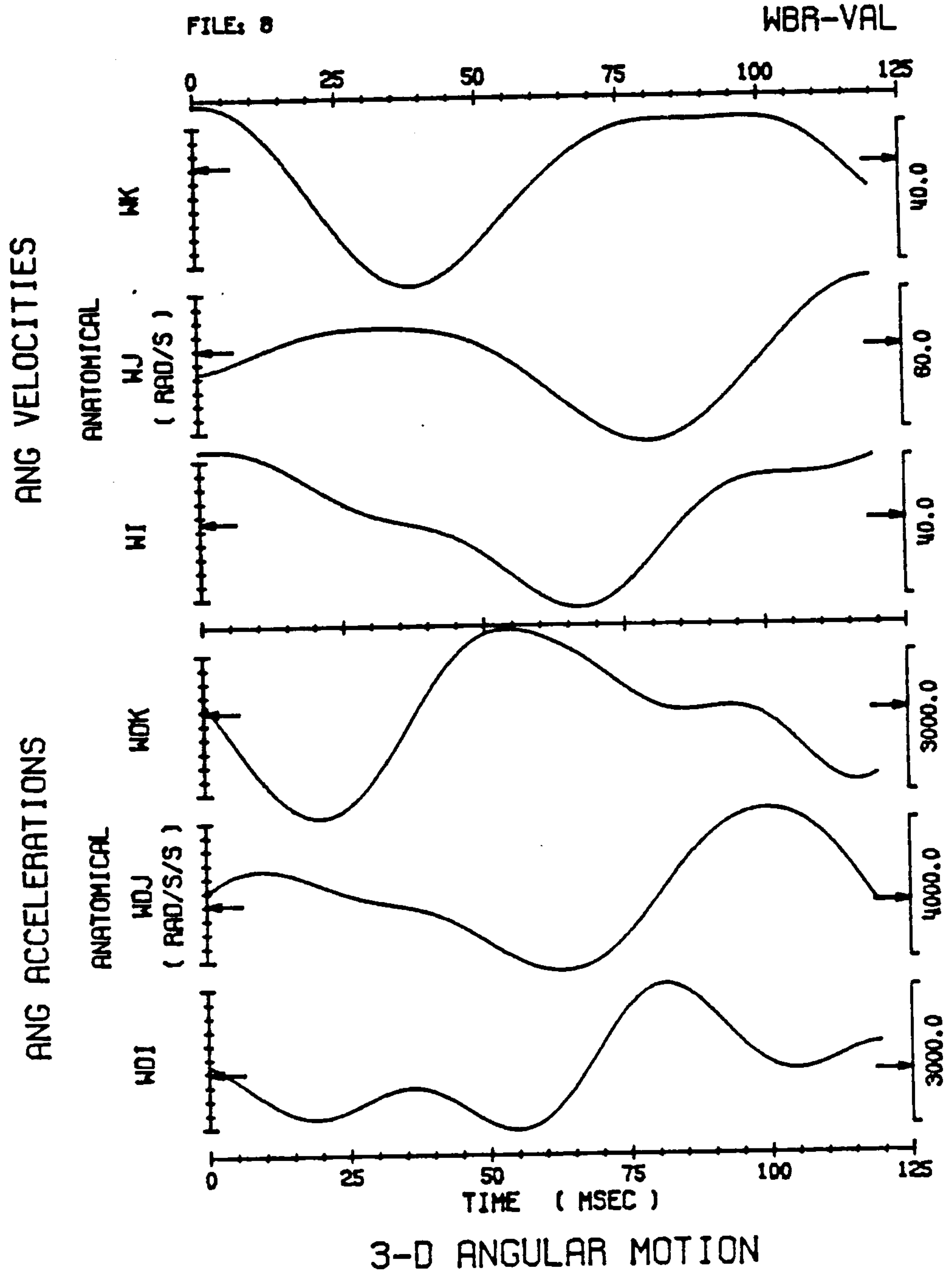


FIGURE 5. HYPOTHETICAL ANGULAR VELOCITIES AND ACCELERATIONS, ALONG THE MOVING ANATOMICAL FRAME.

Validation: Hypothetical Motion

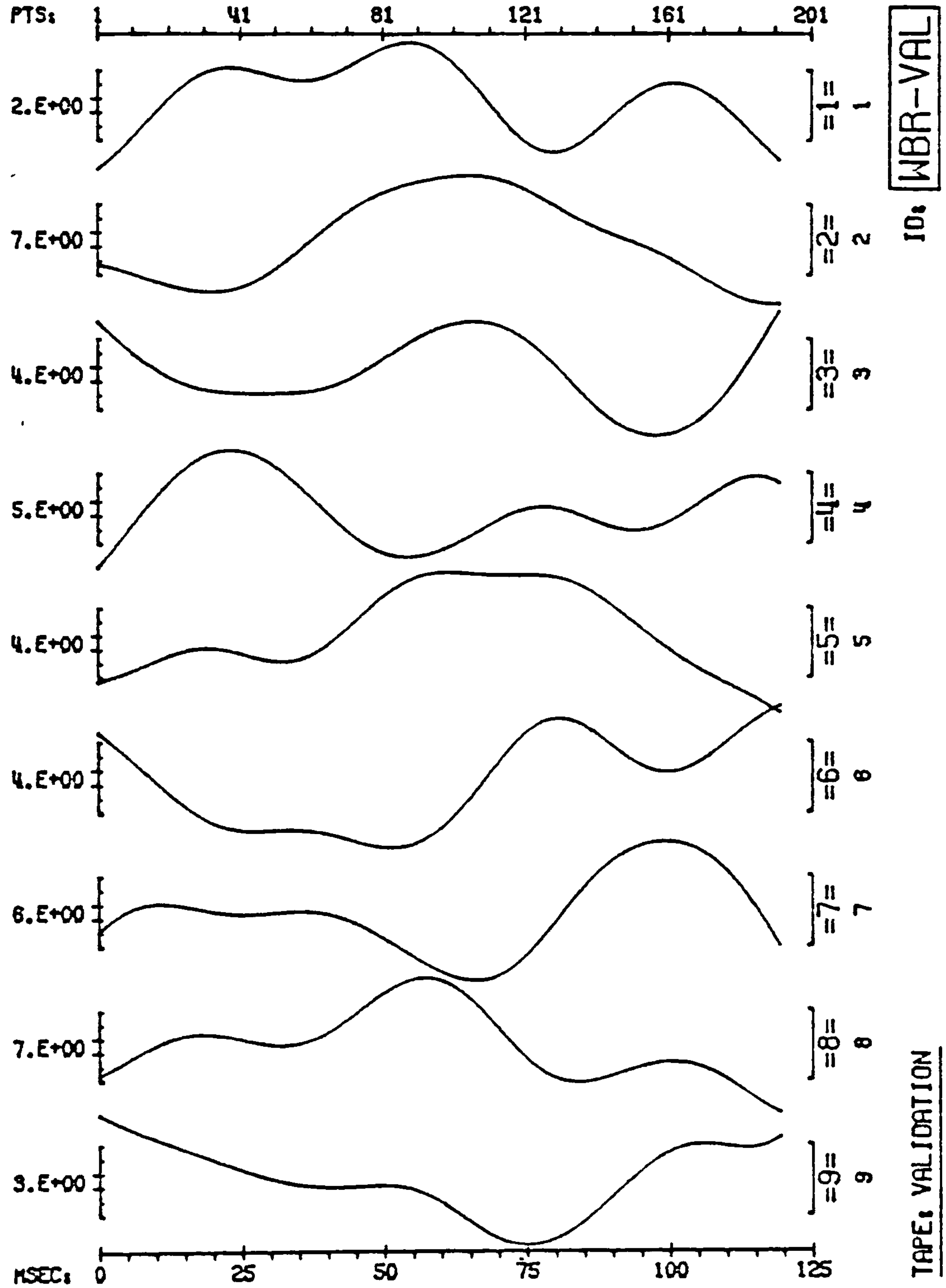


FIGURE 6. SIMULATED READINGS OF NINE ACCELEROMETERS USED AS INPUT TO "NINACC" TO COMPUTE THE 3-D MOTION.

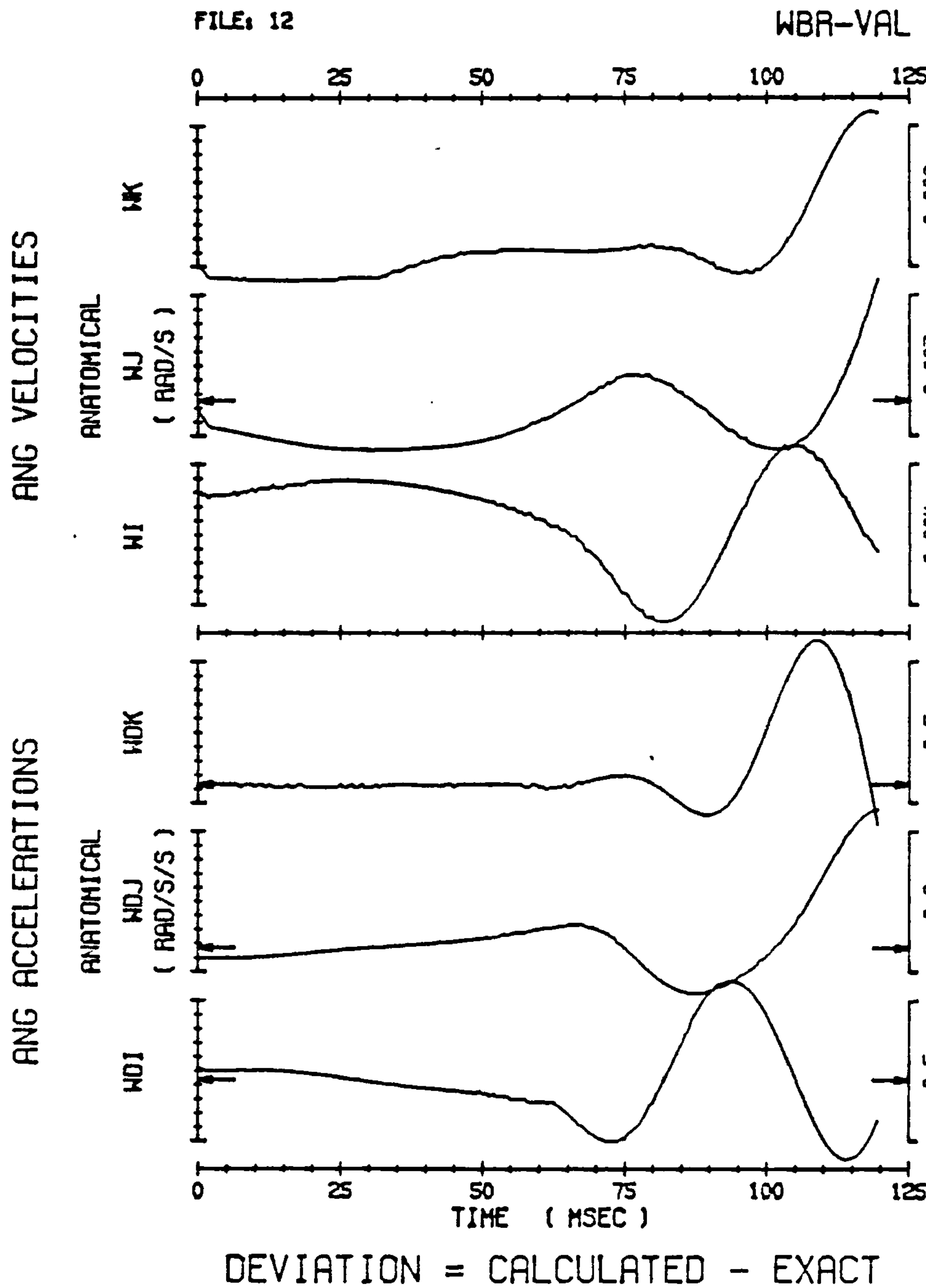


FIGURE 7. DEVIATIONS OF "NINACC" OUTPUT FROM THE EXACT MOTION GIVEN IN FIGURE 5.

Validation: Hypothetical Motion

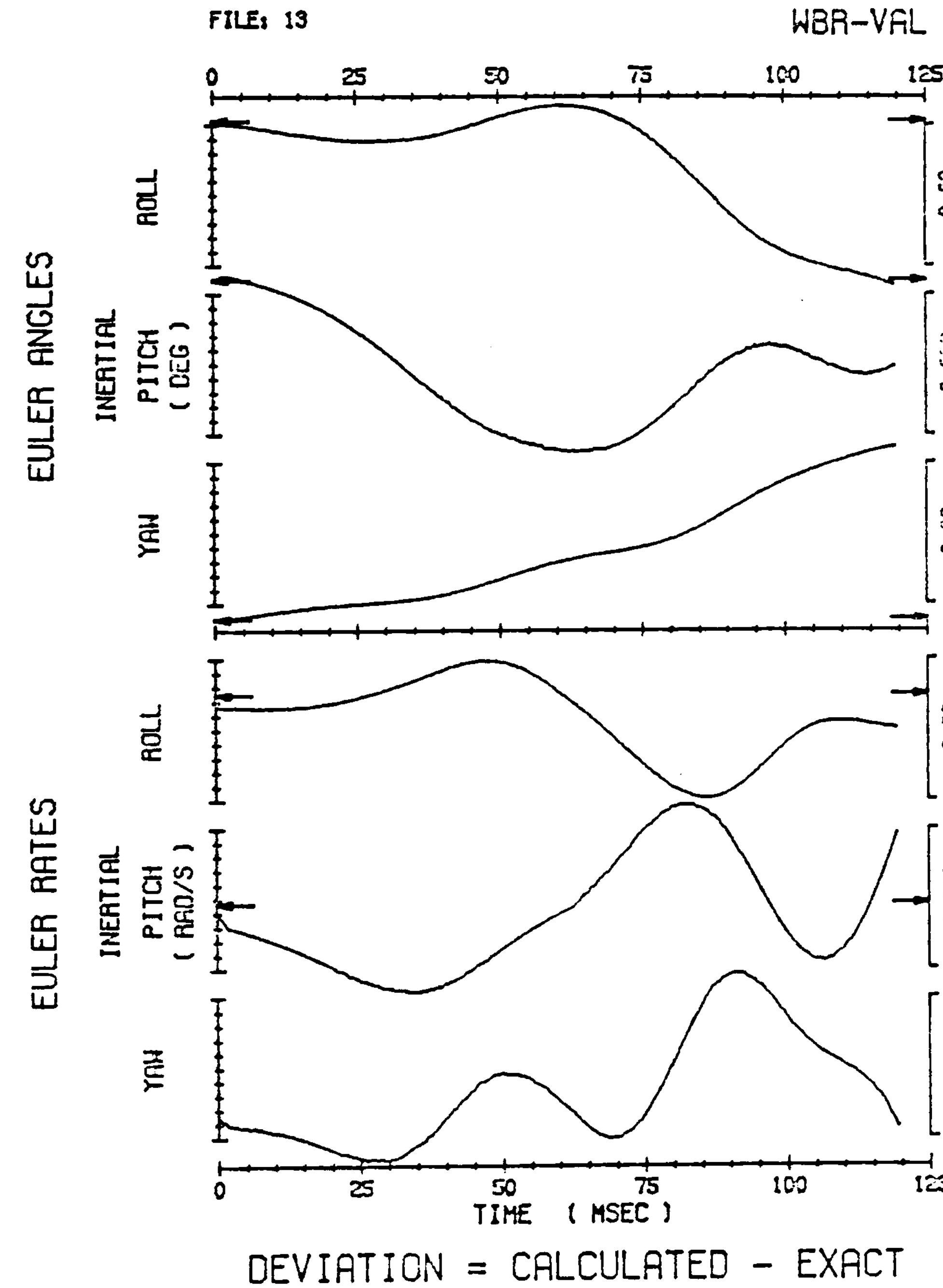


FIGURE 8. DEVIATIONS OF "NINACC" OUTPUT FROM THE EXACT MOTION GIVEN IN FIGURE 4.

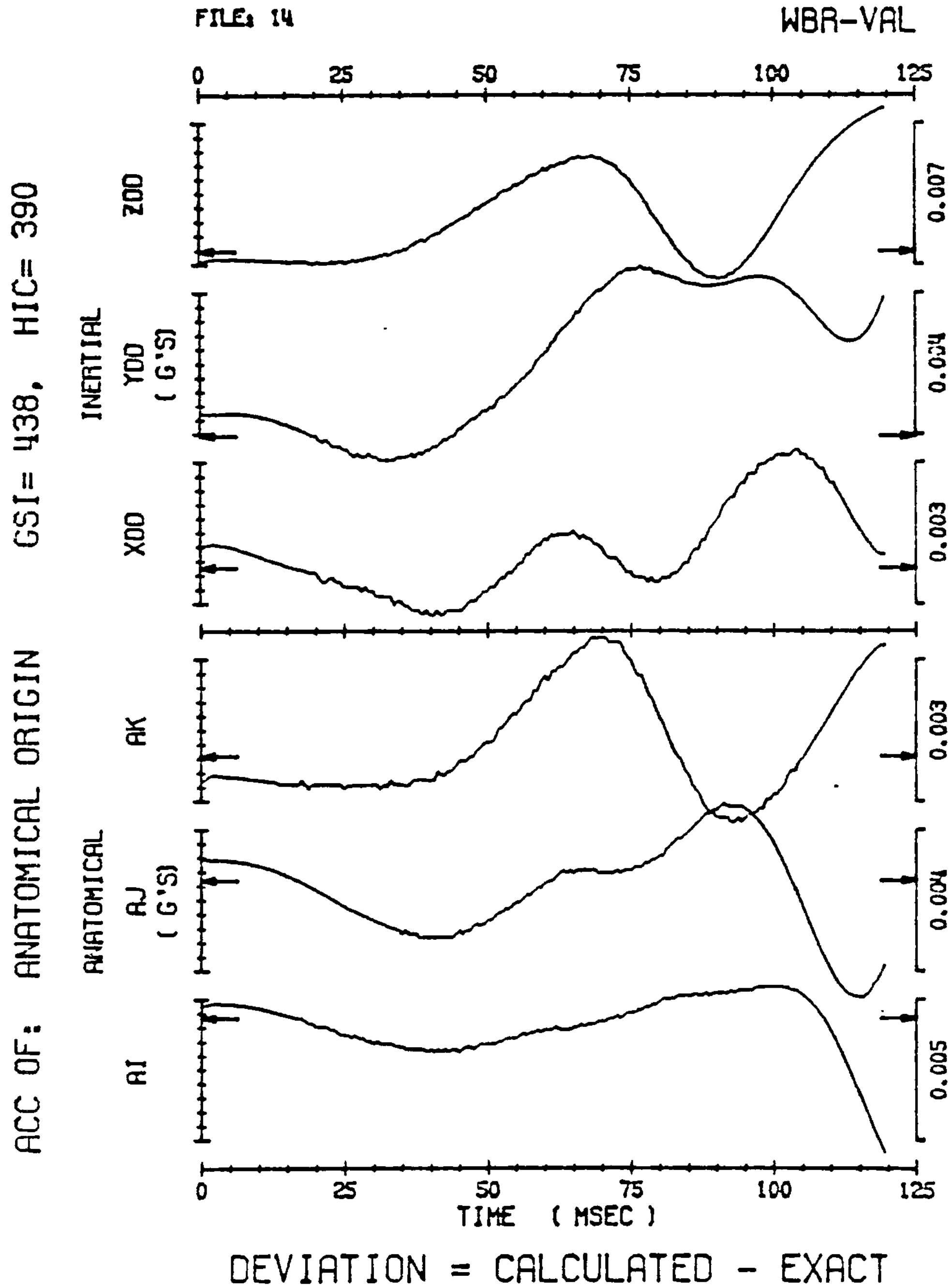


FIGURE 9. DEVIATIONS OF "NINACC" OUTPUT FROM THE EXACT MOTION GIVEN IN FIGURE 3.

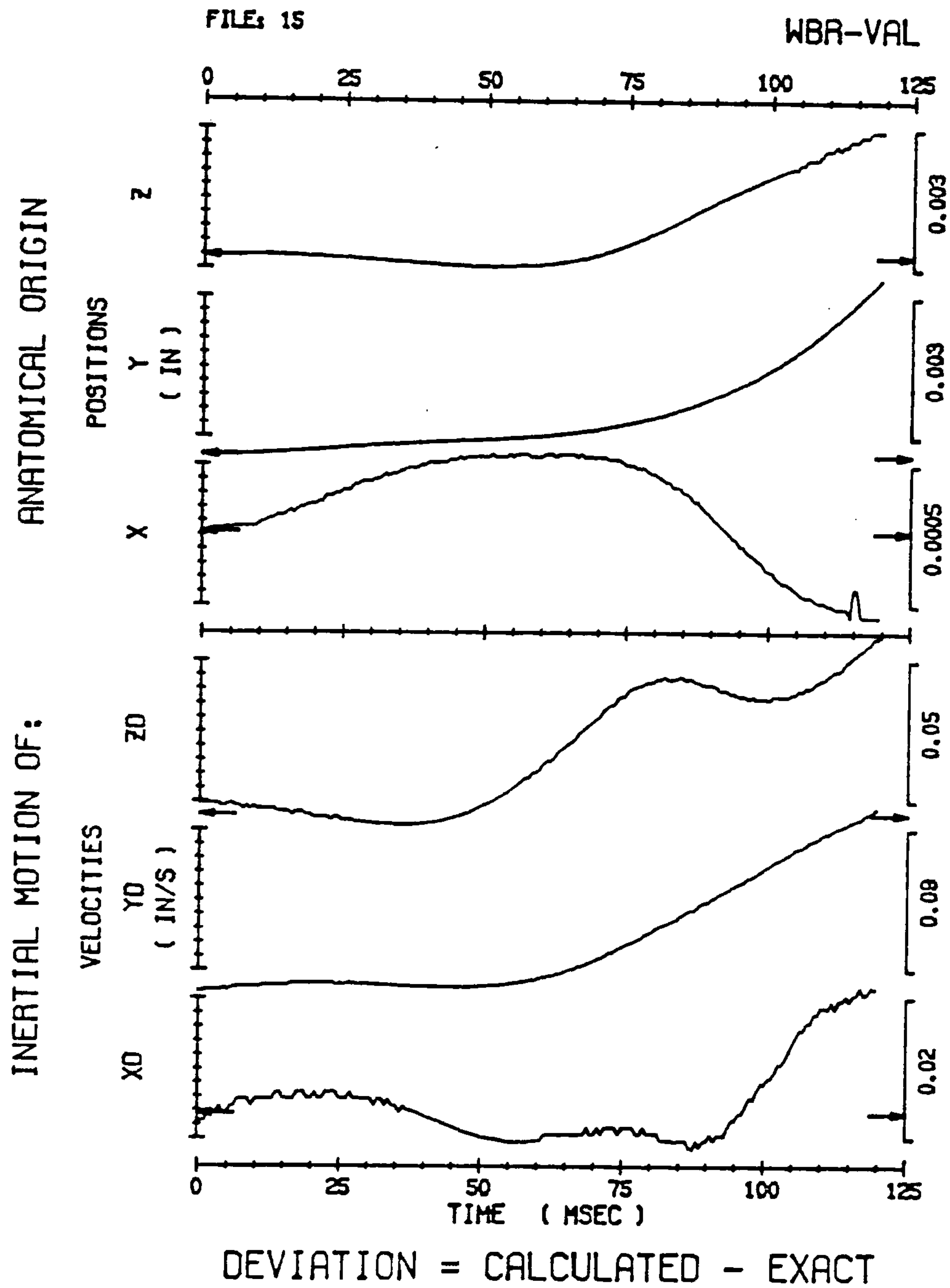


FIGURE 10. DEVIATIONS OF "NINACC" OUTPUT FROM THE EXACT MOTION GIVEN IN FIGURE 2.

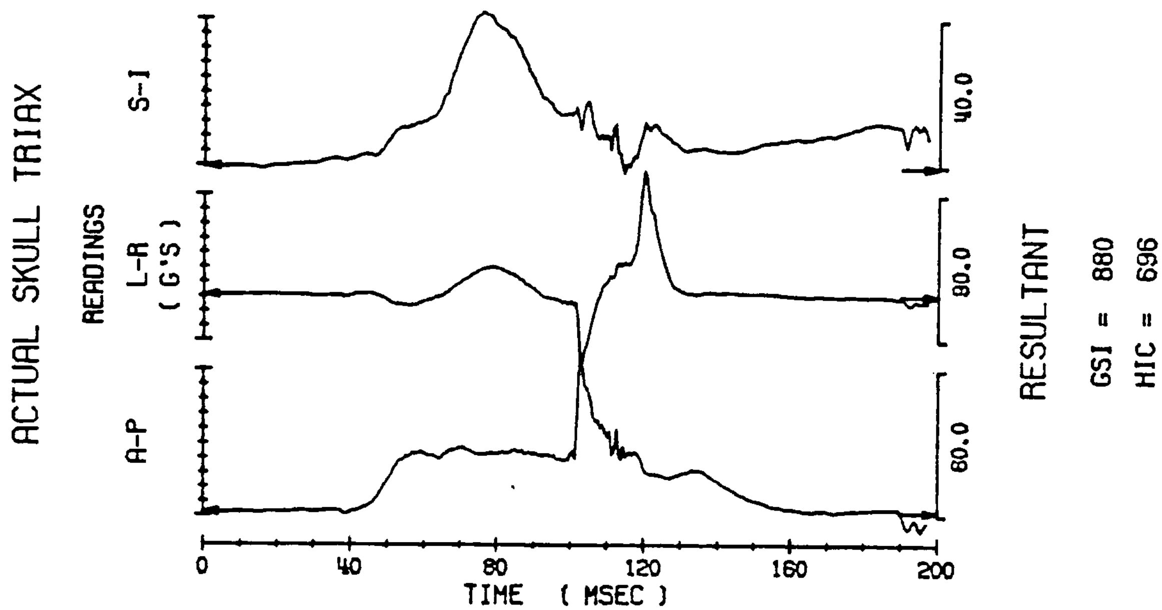
The above hypothetical example represents a 3-D motion with magnitudes of the same order as those encountered in a typical non-impact head motion. In this example, the deviations of the angular accelerations were of the order of 2. rad/s over magnitudes of the order of 3000. rad/s. The ratio of these two numbers may be used as a quantitative measure of the relative accuracy of the analysis, which is of the order of 0.07% for the angular accelerations. Similarly, the orders of accuracy are 0.10% for angular velocities, 0.08% for Euler angles, 0.07% for translational accelerations, 0.10% for velocities, and finally, 0.10% for displacements.

Such accuracies are not typical of engineering measurements because no experimental errors were involved in generating the acceleration readings. The above-descibed deviations result solely from two source: truncation errors due to the finite length of a computer word, and the approximation (round-off) errors in the numerical integration procedure. These are negligeable when compared to the experimental errors associated with actual measurements in real experimental situations.

Validation: Experimental Motion

In order to produce a precisely known, yet general, 3-D rigid body motion in a laboratory, elaborate mechanisms must be constructed to vary simultaneously the six degrees of freedom in a controlled fashion. For validation purposes, it is much simpler to let an instrumented rigid body undergo an unknown general motion and measure the acceleration vector at an arbitrary body-point using a triaxial accelerometer; then, using the 9-accelerometer package, predict the acceleration vector at the same point and in the same moving directions.

Such an experiment was included in a full Whole Body Response sled run, test number 77B001, which was conducted to measure the 3-D motion af an embalmed cadaver head. In addition to the 9-accelerometer package, the head was instrumented with a triaxial accelerometer which was mounted inside the head at the base of the skull, and the brain was substituted by a jelly-like substance. The location and orientation of the triax relative to the head anatomical reference frame were determined using the HSRI 3-D x-ray



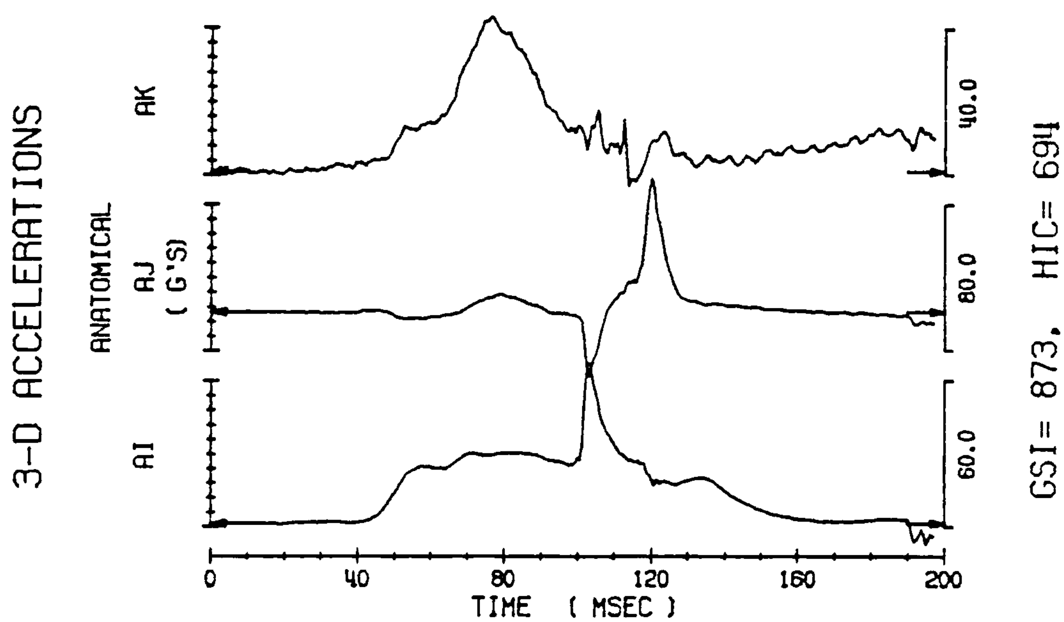


FIGURE 11. ACCELERATIONS AT THE BASE OF THE SKULL, TEST 77BØØ1. TOP 3 CURVES ARE MEASURED DIRECTLY. BOTTOM ONES ARE COMPUTED BY "NINACC."

technique, described elsewhere in this Appendix. After the test was conducted, the 12 transducer signals were digitized and the "NINACC" program was run to predict the acceleration components along the sensitive axes of the triax, and at its center.

These accelerations are shown in figure 11. The top three curves are directly measured with the triaxial accelerometer; the bottom three are indirectly computed by "NINACC" using the 9 acceleration signals. The agreement between the two sets of curves is striking; however, the deviations between the actual and computed ones are significantly larger than those obtained in the hypothetical example of the previous section. Quantitative evaluation of these deviations may be best summarized by computing the HIC and GSI of both resultants. The HIC numbers were 696 for the actual resultant, and 694 for the calculated one. The GSI's were 880 and 873 for the actual and calculated resultants, respectively.

These deviations may be attributed to the errors commited during mounting, x-ray analyses, transducer calibrations and other phases of signal processing. Therefore, it may be concluded that the accuracy of the 3-D motion analysis method is limited by the transducers themselves and the expermimental techniques used to generate the required input to "NINACC."

Limitations Of The Method

An in-depth investigation of the accuracy of this and other 3-D motion measurement methods is underway at HSRI. Some of the influencing factors are discussed briefly in this section. The accuracy of any analysis based on experimental measurements depends mainly on the errors commited during the instrumentation of the experiment. These errors may be divided into 4 major groups, according to their sources and types.

TYPE 1 MISALIGNMENT & MISLOCATION. These errors are due largely to the mechanical mounting of the accelerometers and to the 3-D methods for measuring their locations and orientations, relative to the anatomical frame. Another source of this type of errors is the physical seperation between the seismic masses of the 3 uniaxial elements, which are used to approximate a single triaxial accelerometer. A third source is the manufacturing process where the alignment is usually not guaranteed to be better than within Ø.5 degrees. Finally, the cross-axis sensitivity of the sensing element, an inherent property, result in errors which may be shown to be equivalent to misalignment arrors.

TYPE 2 MIS-CALIBRATION. Most calibration procedures cannot guarantee perfect accuracies in converting volts into physical units. Errors could be made at any point in the measurement chain which includes transducers, signal conditionners, tape recording and playback amplifiers, A-to-D converting unit, and the analog and digital filtering processes.

TYPE 3 NOISE. The transducers' signals contain high frequency components which do not result from rigid body motion per se. These could be internal to the signal processing equipments, such as amplifier noise and cross-talk between channels, or external from mechanical vibrations of the mounts at some of their resonnant frequencies. Most of this noise can be filtered out; however, the lower harmonics and other lower frequencies may be at or below the actual motion's frequencies. In this case, the noise cannot be filtered out without altering the real signal; thus, the noise-to-signal ratio becomes an important factor in the accurate measurement of 3-D motion. If these noise-related errors are random-like in nature, then their effects on the accuracy is minimized by the least-squares method presented earlier.

TYPE 4 ZERO-BIAS. This type of errors result from mis-adjustment of the equipment during calibration, testing, recording, playback and A/D conversion processes. Although this type of error can be delt with by forcing the pre-test portion of the digitized signal to zero, and adjusting the remaining portion accordingly, the problem is not completely solved since a drift in on amplifier can cause an unknown and variable zero shift. Furthermore, an exact 3-D analysis must take into account the acceleration of gravity, which is neglected when all signals are zero-adjusted. This may be tolerated if the motion being measured produces accelerations which are much higher than that of gravity. Otherwise, the initial orientation of each accelerometer, relative to the gravity field, must be known in order to correct for gravity.

It is difficult to point out one source of error that is most damaging to the accuracy of 3-D motion analysis; however, it is safe to state that experimental errors are inevitable, and to conclude that one can only attempt to minimize their effects, first in the laboratory by careful instrumentaion, then during analysis by using a least-squares technique.

Summary

By proper measurements of 9 linear accelerometers embedded in the moving rogid body, the six degrees of freedom of 3-D motion may be completely determined, at the acceleration, velocity and displacement levels. The equations of motion are derived by minimizing the error of measurement, in the least-squares sense. The accuracy of computed motion is limited by the accuracy of the experimental measurements, and the signal processing techniques used to prepare the input to the analysis program.

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